Graphing Rational Functions

(Holes, Vert. & Horiz. Asymptotes)

MathJam Concept Drop

➢ **RATIONAL FUNCTION**: A function that is expressible as a fraction with polynomials in the numerator and the denominator.

- Example: $f(x) = \frac{x^2 - x - 56}{x^2 + 9x + 14}$

☐ Step 1: Factor the numerator and denominator.

- ☐ Example: $f(x) = \frac{x^2 - x - 56}{x^2 + 9x + 14} = \frac{(x+7)(x-8)}{(x+7)((x+2)}$

☐ Step 2: Identify **Holes**.

- ☐ **Holes**: Holes exist when there is a common zero in the numerator and denominator.
  - ☐ Example: This problem has a hole at x = -7.

☐ Step 3: Determine **Vertical Asymptotes**.

- ☐ **Vertical Asymptotes**: Guides for the behavior of a graph as it approaches a vertical line.
  - ☐ Let $P(x)\ and\ Q(x)\ be\ polynomials\ for\ a\ rational\ function\ \frac{P(x)}{Q(x)}$
  - ☐ Vertical asymptote(s) exist for all values of, *a,* for line *x* = *a*, such that $Q(a) = 0$ as long as $P(a) \neq 0$.
  - ☐ Example: $Q(a)$ in the example is (x+7)(x+2). $x = -7\ and\ x = -2$ are zeros, but a **vertical asymptote will only exist at** $x = -2$. Since $x = -7$ is a zero for both the numerator and denominator (where the hole exists), it will not have a vertical asymptote at -2.
  - ☐ The up or down behavior of the function as it approaches the asymptote can be determined by substituting values close to *a* on either side of the asymptote.

☐ Step 2: Determine **Horizontal Asymptotes.**

- ☐ **Horizontal Asymptotes:** Guides for the behavior of a graph as it approaches a horizontal line.
- ☐ 3 situations to consider:
  - ☐ (1) When the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is at y = 0.
    - ☐ Example: $f(x) = \frac{x+4}{x^2 - 25}$

- ☐ (2) When the degree of the numerator is greater than the denominator, there is no horizontal asymptote.
  - ☐ Example: $f(x) = \frac{x^3+2x^2-4x-11}{x^2+7x+12}$
- ☐ (3) When the degree of the numerator is equal to the degree of the denominator, set *y* equal to the ratio of the leading coefficients. The graph of this line is the horizontal asymptote.
  - ☐ Example: $f(x) = \frac{4x^2-2x-12}{2x^2+13x-7}$ Horiz. Asymptote, y=$\frac{4}{2} = 2$