How to find the distance from a given point to a line.

Point: (x_1, y_1) Line in Standard Form: $ax + by + c = 0$

Formula to use to find the distance from a point to a line: $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Remember! The distance is always the shortest path from the point to the line, hence, it creates a perpendicular intersection.

Example: Find the distance between the given point and line.
$(4, 5)\ and\ y = \frac{2}{3}x - 3$

Step 1: Rewrite the equation of the line into standard form (if already given in standard form, then proceed to step 2):

$y = \frac{2}{3}x - 3$: subtract $\frac{2}{3}x$ and add 3 to each side so it is set equal to zero.

$-\frac{2}{3}x + y + 3 = 0$: Multiply all terms on both sides of the equation by -3 to ensure the *a* value is a positive integer.

$2x - 3y - 9 = 0$

Step 2: Identify the variables.

$x_1 = 4,\ y_1 = 5,\ a = 2,\ b = -3,\ c = -9$

Step 3: Substitute the values into the distance formula.

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \Rightarrow \frac{|(2)(4)+(-3)(5)+(-9)|}{\sqrt{(2)^2+(-3)^2}} \Rightarrow \frac{|8-15-9|}{\sqrt{4+9}} \Rightarrow \frac{|-16|}{\sqrt{13}} \Rightarrow \frac{16}{\sqrt{13}}$$

Because this problem has a radical in the denominator, we must rationalize the denominator.

$$\Rightarrow \frac{16}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} \Rightarrow \frac{16\sqrt{13}}{13}$$

Practice Problems

Find the distance between the given point and line.

1) $(3, 4)\ and\ y = \frac{3}{4}x - 2$

2) $(4, 4)\ and\ y = \frac{-4}{3}x - 1$

3) $(0, 2)\ and\ y = x$

4) (1,−2) and y=−x−6

5) (−3,4) and y=−12x−2